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ONTARIO

DEPARTMENT OF EDUCATION

MATHEMATICS

Intermediate Division

Grade 9

Five-Year Program

Four-Year Program

For use in Grade 9 for students who have completed Curriculum I.12A and I.12B

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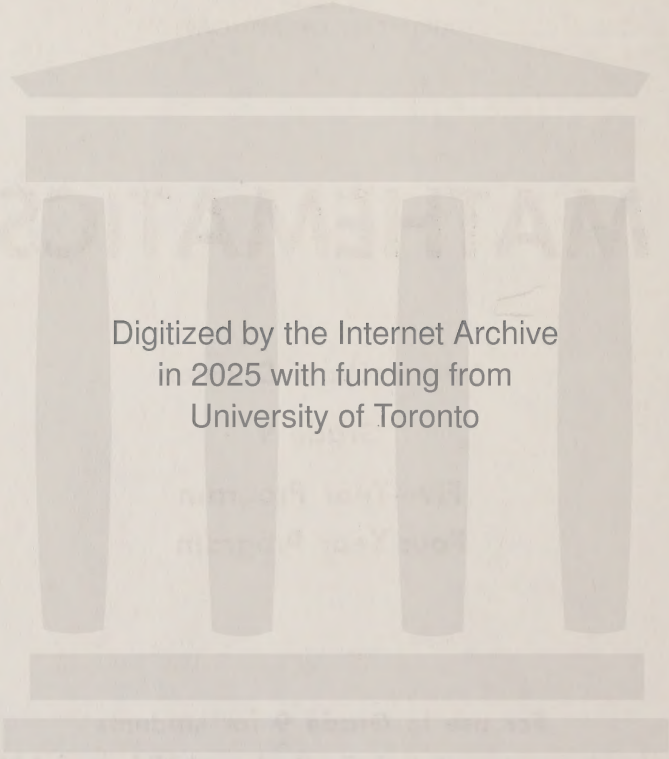

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MATHEMATICS

Grade 9: Five-Year Program

Introduction

This course is designed for pupils of the five-year program in Grade 9 who have completed successfully both of the earlier courses of this series, that is, Curriculum I.12A (Grade 7) and Curriculum I.12B (Grade 8). Use for any other pupils must take account of the sections of these courses not previously studied.

Unit 1: The Whole Numbers and the Integers

(3 to 4 Weeks)

The Grade 7 course provided a development of the structure of the whole number system and practice with the whole numbers. The Grade 8 course continued this development and introduced the integers. This first unit of Grade 9 is to provide a recall and a review of these two topics.

Except for the introduction of the concept of variable, the work of the whole numbers will be mainly of a recall nature. In the case of the integers, however, it would be expected that considerable practice will be required in the operations as well as with the new topic of absolute value. It should also be noted that in the work on the integers in Grade 8, the raised signs were used throughout. As a result, pupils will require practice to become familiar with the signs in the regular position. It is therefore suggested that the work on the integers should receive the greater part of the time devoted to the unit.

1. The Whole Numbers

   (a) The concept of variable in number phrases or expressions and in number sentences

   Solution by inspection of number sentences involving equalities and inequalities where the variables represent whole numbers

   Graphing of solution sets

(b) The system of whole numbers

The closure, commutative, associative, and distributive properties in the system

The identification of zero and one as identity (neutral) elements for addition and multiplication respectively

2. The Integers

(a) The system of integers

The property of opposites

Addition, subtraction, multiplication, and division of integers

The closure, commutative, associative, and distributive properties in the system

The identity and inverse elements

(b) Specific examples of the absolute value of integers, e.g., $|-5|$, $|+3|$

(c) Solution by inspection of number sentences where the variables represent integers

Graphing of solution sets

Unit 2: The Real Numbers

(4 Weeks)

The Grade 7 course provided an introduction to the system of fractional numbers, the Grade 8 course extended the concept slightly, and this unit extends it to establish the rational numbers as arising when the two elements involved are integers.

Some practice is then introduced in changing numerals of the form $\frac{a}{b}$ to decimal notation and from decimal notation to the form $\frac{a}{b}$ in order to establish a recognition of one-to-one correspondence between the rational numbers and the terminating or repeating decimals, and to point out the existence of non-terminating, non-repeating decimals representing a different set of numbers called the irrationals.

It is not intended to introduce an exhaustive treatment of the irrationals. Some simple computation is introduced with square roots but the basic aim in the introduction of irrationals is to reach the stage where the property of one-to-one correspondence between the real numbers and points on a line may be assumed.

1. The Rational Numbers

   (a) Definition of a rational number as any number that may be expressed in the form $\frac{a}{b}$ where a and b are integers and b is not equal to zero, and where $\frac{a}{b}$ means a divided by b

   (b) Discussion of the operations in the set of rational numbers

   Transformation of simple rational number expressions using the commutative, associative, distributive, and closure properties

   (c) Practice at changing numerals of the form $\frac{a}{b}$ to decimal notation and an inductive development of the fact that these decimals either terminate or repeat

   Practice at changing terminating decimals to the form $\frac{a}{b}$ in order to indicate that this change is always possible

   Illustration with a few simple cases of the method of changing repeating decimals to the form $\frac{a}{b}$ in order to indicate that this change appears to be always possible

   (d) Illustration of the fact that there is a rational number between every pair of distinct rational numbers

2. The Irrational Numbers

   (a) Reference to the existence of a different kind of number corresponding to the non-terminating, non-repeating decimals

Reference to numbers of this type already encountered e.g., π

(b) Demonstration that the irrational numbers may be identified with points on a number line

3. The Real Numbers

(a) Identification as the union of the rationals and the irrationals

(b) Assumption of the property of one-to-one correspondence between the real numbers and points on a number line

(c) Specific examples of the absolute values of real numbers (numerical cases only)

(d) A recapitulation of the various number systems up to and including the reals

Unit 3: Number Sentences of the First Degree

(4 to 5 Weeks)

In previous grades and to this stage in Grade 9, number sentences involving equalities and inequalities have been solved by inspection. The first objective of this unit is to develop the solution of number sentences using the number properties. The second objective is to give practice in the manipulation of formulas, treating them as number sentences to be solved using the number properties. The third objective is to give practice in the use of number sentences in the solution of problems, including those arising in science and in commercial and technical work.

1. The solution of number sentences

(a) Review of the solution of number sentences by inspection and the graphing of the solution sets on a number line

(b) Solution, using the number properties, of number sentences involving equalities and inequalities

(c) Practice in transforming formulas arising in science, mensuration, and business arithmetic by making use of the number properties

2. The solution of problems

(a) Analysis of problems

(b) Formal written solutions to include a statement of any limitations on the variable assigned

Unit 4: Exponents and Related Topics

(3 Weeks)

The Grade 7 course outline included an introduction to exponents and the Grade 8 course provided some further practice. It may be well, however, to assume that, *for the most part,* this practice will have been limited to cases where the exponents were natural numbers greater than 1 and the bases were whole numbers or fractional numbers. Similarly, it should be noted that the calculation of square roots in Grade 8 was limited for the most part to smaller natural numbers and that the method of estimation was used for those that were not perfect squares. In this Grade 9 course outline, it is to be noted that Newton's method of determining square roots replaces the traditional method.

1. Extension of the practice in simplifying number expressions of one or more terms involving exponents to include cases where the bases are rational numbers and the exponents are integers

   Practice with the exponent rules for multiplication and division when the exponents are integers

   Practice in writing very large and very small numbers in scientific (or standard) notation and some use of conversions to scientific notation in estimating products and quotients

2. Definition of the principal square root of a real number (numerical cases only)

   Some practice in operations with principal square roots restricted to multiplication, second powers, addition and subtraction and simple transformations such as $\sqrt{8} = 2\sqrt{2}$, $\sqrt{12} = 2\sqrt{3}$

   Use of these relationships to find several square roots, e.g., $\sqrt{8}$, $\sqrt{12}$, $\sqrt{18}$, having been given numerical values for $\sqrt{2}$ and $\sqrt{3}$

3. Use of Newton's method of determining square roots

4. Use of a square root table in problems such as those involving the Pythagorean relationship in right-angled triangles (Grade 8, Unit 6), and the radii of circles or the sides of squares where areas are given

   Some practice with formulas involving square roots and simple powers

Unit 5: Polynomial and Rational Expressions

(7 Weeks)

This unit begins with the study of the values of an expression for various values of the variable. Factoring of polynomials is then studied for use in the operations with rational expressions.

1. Practice in establishing tables of values of expressions such as x^2, $2a^3 + 3$, $\frac{1}{z-5}$ for various values of the variable, such practice to include the use of a symbol such as E(x) to represent either a polynomial or rational expression

   Recognition of specific values of the variable for which no value of the rational expression exists

   (Supplementary) Study of the values of the expression for values of the variable close to these critical values

2. Practice in addition, subtraction, multiplication, and division of polynomial expressions (including long division)

   Recognition of limitations necessary in the divisor

3. A study of polynomials with rational coefficients to establish

   (i) The factors of monomials

   (ii) The square roots of a monomial

   The principal square root of a monomial, e.g. $\sqrt{x^2} = |x|$

   (iii) The factors of binomials involving common factors (as an application of the distributive property)

   (iv) The squares of binomials

   (v) The square roots of a trinomial that is the square of a binomial

   The principal square root of a trinomial, e.g., $\sqrt{x^2 - 4x + 4} = |x-2|$

   (vi) The factors of trinomials (including the use of the decomposition method)

(vii) The factors of an expression that is the difference of two squares

4. Applications of factoring to the multiplication, division, addition, and subtraction of rational expressions

Unit 6: Geometry and Measurement

(8 Weeks)

The course outlines for Grades 7 and 8 provide an introduction to the concepts and simple properties of some of the geometrical figures such as point, curve, line segment, ray, plane, polygon, angle, and circle, and some practice at ruler-and-compass construction associated with these figures. The course outlines also provide for the discovery of some geometrical relationships such as the angle-sum of a triangle, the equality of angles in an isosceles triangle, the equalities of alternate and corresponding angles in parallel line configurations, and the Pythagorean relationship for right-angled triangles. In addition, these course outlines provide for the discovery that certain particular combinations of measures (three sides, one side and two adjacent angles, two sides and the contained angle) are sufficient to determine one and only one triangle.

Throughout Grades 7 and 8 discovery using the inductive method has been stressed. The Grade 8 course outline actually includes a suggestion which unites the inductive method with the scientific method by specifying steps in the discovery approach (recognition of a possible relation, conjecture leading to a hypothesis, testing of the hypothesis, statement of the theory).

This unit in Grade 9 is organized to make use of the same inductive method in the discovery of the more commonly-used geometrical relationships associated with angles, triangles, quadrilaterals, and circles.

The intention is that formal deductive geometry should be reserved for introduction in Grade 10, although this intent is not to preclude the use of simple common-sense deduction in the numerical problems that will follow as applications of the facts discovered inductively. Section 6 of the unit provides an introduction to the concept and the use of ordered pairs of numbers.

1. A review of the properties of the geometrical figures studied in previous grades with emphasis on the equality relationships associated with these figures

   A review (from Grade 8, Unit 6, Part 2) of the six standard ruler-and-compasses constructions relating to angles and to parallel and perpendicular lines

   Practice with the language used in description of constructions

2. Inductive discovery of the equality relationship between the exterior angles of a triangle and the two interior and opposite angles

   Numerical problems using this relationship

3. Inductive discovery of the area relationships involving triangles and parallelograms

   Numerical problems using these relationships

4. (a) A development of the concept of congruency related to line segments, angles, and triangles, stressing the use of one-to-one correspondence

   (b) A review of the particular combinations of equalities giving one (and only one) triangle in constructions

   Conjecture as to the minimum conditions sufficient for congruency of two triangles

   Testing of the conjecture using several examples

   Consideration of the ambiguous case

   (c) Practice in writing congruence correspondences from various pairs of triangles

   (d) An inductive development of equal ratios in similar triangles

   Numerical examples applying these relationships

5. Inductive discovery of the following relationships in the circle (and the development of the necessary concepts associated with the relationships)

   (i) The centre of a circle lies on the right bisector of any chord

   (ii) The line through the centre of a circle and the mid-point of a chord is perpendicular to the chord

(iii) The measure of an angle inscribed in a circle is equal to one half that of one at the centre standing on the same arc

(iv) Angles inscribed in a circle that are subtended by the same arc are equal in measure

(v) An angle inscribed in a semicircle is a right angle

(vi) (Supplementary) The angle between a radius and a tangent is a right angle

(vii) (Supplementary) Tangents to a circle from a point are equal in measure

Numerical problems using these relationships

6. (a) A development of the concept of ordered pairs arising from the combining of elements of two sets

Graphing of ordered pairs

Practice in plotting sets of ordered pairs that determine various rectilinear figures such as lines, line segments, rays, triangles, and quadrilaterals

(b) A review (from Grade 8, Unit 5, Part 3) of the rate graph to show that these line graphs are the graphs of sets of ordered pairs

Graphing of the sets of ordered pairs given by $y = x$, $y = 2x$, $y = 3x$ (x and y real numbers)

(Supplementary) A discussion of the difference in form of a rate graph if the units involved in the cost are, for example, marbles (instead of gallons)

(c) Length and slope of the line segment joining two points (particular cases only)

Easy numerical problems involving slopes and lengths of line segments

Unit 7: Enrichment Mathematics (Optional)

The first six units of this course provide what might be described as the main stream of mathematics for grade 9 pupils. The topics there listed provide all the necessary development in preparation for mathematics of higher grades. However, in these days of widening interest in mathematics, pupils are finding great interest and excitement in exploring the bays and inlets along the main

stream of mathematics. This unit lists *some* of the topics which form these bays which some pupils will delight in exploring.

It is not the intention that all pupils of a class would study all of a topic, although this might conceivably occur. Rather, it might be that after interest is aroused, some pupil or pupils might be encouraged to pioneer their own ways in and around the inlets of knowledge associated with a topic, and perhaps report back to the teacher or class or develop some project suggested by their discoveries.

Although the regular text-books may introduce the student to the study of some of these topics, it is not expected that they will treat any of them exhaustively. Rather, as an introduction to the techniques of exploration in depth, it is better if the pupils have to depend on other source materials. In these days of a proliferation of pamphlets and books available for libraries on specific mathematics topics suitable for different levels, no great difficulty should be encountered by the teacher in this one problem associated with the encouragement of search.

Some Possible Topics for this Unit

1. An introduction to the mathematics of the computer
2. Three-dimensional models
3. Translation, reflection, and rotation of regular polygons
4. Experiments in probability and statistics
5. Modular arithmetic
6. An introduction to mathematical systems
7. Patterns in number
8. Vectors and navigational problems
9. Linear programming
10. Selected topics from the history of mathematics

MATHEMATICS

Grade 9: Four-Year Program

Introduction

This course is designed for the pupils of the Four-Year Program in Grade 9 who have completed both of the earlier courses of this series, that is, Curriculum I.12A (Grade 7) and Curriculum I.12B (Grade 8). Use for any other pupils must take account of the sections of these courses not previously studied.

It is the aim of this course to prepare pupils for the subsequent mathematics of the Four-Year Arts and Science, Business and Commerce, and Science, Technology and Trades Branches, and at the same time, to develop an understanding of basic mathematical concepts.

Throughout the course, liberal use of practical problems from business and industry is made. The necessary drill in arithmetic is achieved through the evaluation of simple algebraic expressions for various replacement sets rather than through the use of "drill" exercises.

Unit 1: The Whole Numbers; $\{0, 1, 2, 3, \ldots\}$

(2 Weeks)

The Grade 7 and 8 courses provide a development of the structure of the set of whole numbers and practice with operations involving whole numbers. This first unit of Grade 9 is to provide a recall and review of this topic.

1. Review of the properties of the system of whole numbers

   Review of the terms associated with the operations of addition, subtraction, multiplication and division

   Practice in addition, subtraction, multiplication, and division achieved by means of the evaluation of first degree algebraic expressions, e.g. $2x + 3y$, $3xy$, $4(x + y)$, $3(2x + 5y)$, $\frac{3x + y}{2xy}$, where the replacement set is the set of whole numbers

   Use of commutative, associative, and distributive properties as aids to computation

Estimation of products and quotients and use in checking answers

2. Review and use of whole number exponents greater than one

   Evaluation of algebraic expressions involving powers e.g., x^2, $3x^2$, $(2x)^2$, $4x^2 + 3y^2$, $x^2 + 12x + 20$, where the replacement set is the set of whole numbers

3. Solution by inspection of number sentences involving equalities and inequalities where the replacement set is the set of whole numbers

   Graphs of solution sets

4. Translation of stated problems into number sentences

   Solution of problems involving number sentences restricted to whole numbers

Unit 2: The Integers

(3 Weeks)

In the work on the integers in Grade 8, raised signs were used throughout; in this course signs in the regular position are to be introduced and used.

1. Review of the properties of the system of integers introducing the common notation with respect to signs
2. Evaluation of algebraic expressions such as $5x$, $3x^2$, $4x^3$, $3x - 5y$, $4x^2 - 3x + 2$, $2x(3x^2 + 6x)$ where the replacement set is the set of integers
3. Solution by inspection of number sentences (involving equalities and inequalities) where the replacement set is the set of integers

   Graphs of solution sets

4. Problems solved by use of number sentences where the solutions may be integers

Unit 3: The Set of Rational Numbers

(3 Weeks)

1. Definition of a rational number as any number that may be expressed in the form $\frac{a}{b}$ where a and b are integers and b is not equal to zero, and where $\frac{a}{b}$ means a divided by b

   Discussion of the operations in the set of rational numbers

   Transformation of simple rational number expressions using the commutative, associative, distributive, and closure properties

   Practice at changing numerals of the form $\frac{a}{b}$ to decimal notation and an inductive development of the fact that these decimals either terminate or repeat

   Practice at changing terminating decimals to the form $\frac{a}{b}$ in order to indicate that this change is always possible

   Illustration of the fact that there is a rational number between every pair of distinct rational numbers

   Simplification of complex fractions by the use of the multiplication by $\frac{x}{x}$ where x is a common multiple of the denominators involved

2. Evaluation of algebraic expressions where the replacement set is the set of rational numbers in either fractional or decimal form,
   e.g., $3x - 5y$, $2xy$, $\frac{x}{y}$, $\frac{3x}{5y} + \frac{2x}{3y}$, $4x^2 - 3y^2$

3. Review of expanded notation, e.g., 4356.327 written as
   $4\times10^3 + 3\times10^2 + 5\times10 + 6\times1 + 3\times\frac{1}{10} + 2\times\frac{1}{10^2} + 7\times\frac{1}{10^3}$

   Use of the idea of the integers and the pattern of decreasing value of the exponents to establish the equivalent form

$4\times10^{+3}+3\times10^{+2}+5\times10^{+1}+6\times10^{0}+$
$3\times10^{-1}+2\times10^{-2}+7\times10^{-3}$

Intuitive determination of a meaning for each of 10^{1}, 10^{0}, 10^{-1}, 10^{-2}, 10^{-3} from these two equivalent forms

Previous experience with scientific (standard) notation extended to numbers less than unity

Estimating products and quotients of numbers in standard notation

Unit 4: Square Roots

(2 Weeks)

1. Review of square roots of whole numbers by estimation and by factoring
   Square roots of whole numbers by the Newton method
2. Square roots of numbers in fractional notation where both numerator and denominator are squares of whole numbers

   Square roots of numbers in decimal notation using the Newton method
3. Use of square root table in problems involving square roots, e.g., the sides of squares where the areas are given, the sides of right triangles
4. Reference to the existence of irrational numbers
5. A recapitulation of the number systems through the whole numbers, the integers, and the rationals, to show
   (a) closure limitations of each system
   (b) types of equations which can be solved

Unit 5: Equations, Formulas and Problems

(7 Weeks)

1. Review of the solution of equations and inequations by inspection

   Graphs of solution sets

   Development of the formal method of solving equations using the number properties and the properties of the equality relation

Practice in solving equations

2. Solution of problems using number sentences extended to illustrate the formal steps in a written solution
3. Practice in the use of formulas and the solution of problems of the following types:
   (i) simple and compound interest and instalment buying
   (ii) insurance and taxes
   (iii) ratio, e.g., mechanical advantages, mixtures, partnerships

Unit 6: Polynomial Expressions

(4 Weeks)

It is intended that this unit will give the pupil an introduction to the algebra of polynomials. The multiplication and factoring of polynomials is to be approached as an application of the distributive property. The examples of factoring should be simple.

1. Addition and subtraction of polynomials
2. Establishment of the exponent rules for multiplication and division when the exponents are positive integers
3. Product of a polynomial and a monomial as an application of the distributive property and recognition of the use of this in computation,
   e.g., $3 \times 43 = 3 \times 40 + 3 \times 3$

   Factors of binomials involving common factors
4. Product of two binomials and the square of a binomial, and applications to computation, e.g.,
   $43 \times 27 = (40 + 3)\ (20 + 7)$ and $(24)^2 = (20 + 4)^2$

   Factors of trinomials where the coefficient of the second degree term is 1
5. Product of the sum and difference of two numbers and its application to computation,
   e.g., $97 \times 103 = (100 - 3)\ (100 + 3)$

   Factors of the difference of two squares

Unit 7: Mensuration

(2 to 3 Weeks)

The items in this unit need not be taught as a separate topic. Some or all of the items may be integrated with Units 5 and 8. Stress should be laid on the drawing of three-dimensional figures.

1. Review of the study of prisms and cylinders
2. Use of the Pythagorean relation to find
   (a) the diagonal of a rectangular solid
   (b) the slant height of a rectangular pyramid and cone
3. Volumes and areas associated with pyramids, cones and spheres

Unit 8: Geometry and Application

(3 to 4 Weeks)

1. Reference to the six standard ruler-and-compasses constructions (reasonably precise oral descriptions to be required)

   Use of these constructions to construct:

   (a) a regular hexagon
   (b) angles of 60° and 45° and constructions of various angles less than 180° which are multiples of 15° and $22\frac{1}{2}°$
   (c) a perpendicular at the end of a line segment without extending the line segment
2. A review of the properties of the geometrical figures studied in previous grades with emphasis on the equality relationships associated with these figures
3. An inductive development of equal ratios in similar triangles

   Problems on ratio in similar triangles, e.g., shadow-heights, road gradients
4. An inductive study of the circle to discover the more important properties associated with the circle, e.g.,

inscribed angles, angle inscribed in a semicircle, centre on the right bisector of a chord, tangents

Construction problems such as finding the centre of a circular disc, drawing a right triangle when given the lengths of the hypotenuse and one side

Unit 9: Two-dimensional Graphing

(3 Weeks)

The purpose of this unit is to discover the form of the graphs and to apply them to practical problems, rather than to study the properties of the graphs.

1. Sets of ordered pairs and their graphs
2. Graphs of sets of ordered pairs determined by equations of the form $y = ax$, $y = ax + b$, $y + ax^2$ and $y = \frac{a}{x}$ for particular integral values of a and b
3. Graphs determined by linear and quadratic formulas
4. Problems solved using graphs, e.g., distance - rate, distance - time, rate - time, cost - number of articles